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Automated numerical estimation of meander length and amplitude

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Abstract: This paper details a novel numerical method for automatic estimation of stream meander length. It is based upon Langbein *et al.* (1966) and produces a single meander length and meander amplitude over a given reach that minimises the least-squares error. This algorithm has been applied to 1,456 reach segments, each approximately of 1 km flow path, throughout the Murray-Darling Basin. These estimates were used in the Physical Form theme of the Murray-Darling Basin Sustainable Rivers Audit (SRA). This paper details the method and presents results from one reach and then a summary from all 1,546 study reaches.

The algorithm for estimation of meander length is based upon the theory of Langbein *et al.* (1966) and further applied and assessed by Williams (1986). The theory proposes that the angle between the flow direction at a given point and the regional stream flow path changes, with reach distance, to produce a sinusoidal relationship. By numerically fitting a sine function to this relationship, a wave length and thus the meander length can be estimated. However, a challenge in applying this method is in defining the regional stream flow path to which the observed angle is calculated. If it was to be applied to only a single reach, the regional flow path could simply be qualitatively estimated by the practitioner. However, implementation to the 1,456 reaches investigated herein required an automatic method for estimation of this regional flow path and estimation of the meander length. To achieve this, and after trialling a considerable number of alternatives, an algorithm was developed to rotate the reach, fit a k order polynomial to estimate the regional flow path, calculate the residuals to this path and then numerically fit the Langbein *et al.* (1966) sinusoidal model to the residuals. The fitting and rotation was undertaken by multi-start Trust-Region non-linear least-squares regression.

Overall, the algorithm developed provided a robust, objective and reproducible means for estimating meander length. Furthermore, those reaches for which the algorithm does not perform satisfactorily can easily be identified by the low coefficient of efficiency. The algorithm does, however, have some weakness. Most notably, the modelled meander length is a single best estimate, in a least-squares sense, to the entire one kilometre reach. If smaller or longer reach chainage were investigated it is likely that the estimate would differ. Therefore, the estimated meander length herein is very likely to be a function of the scale of the application.

Keywords: *Meander wavelength, meander amplitude, Murray-Darling Basin, River planform, River health, Channel change*

1. INTRODUCTION

The Sustainable Rivers Audit (SRA) for the Murray-Darling Basin (MDB) (Davies *et al.*, 2008) is a three-yearly report on the status and trend in ecological condition of the basin's riverine environment. The SRA assessment addresses five theme areas: macro-invertebrates, fish, vegetation, hydrology and physical form. Physical form refers to river morphology and the underpinning fluvial processes. Fluvial morphology is modified by human disturbances and in particular changes in stream sediment and flow regimes. Such changes result from activities such as land clearing, mining, water resource developments and stock access to the water. These changes are ubiquitous across the MDB and consequent channel changes have occurred in upland and lowland rivers including changes in channel planform, cross-sectional dimensions and channel variability. The physical form theme assesses these conditions by comparing observed channel geometry with the channel geometry that might be expected in the absence of European settlement. This expected or reference state channel geometry is modelled. Observations of the current geometry were made using airborne LiDAR (Light Detecting and Ranging) of 1,456 river reaches randomly selected across the MDB. The LiDAR data was then processed to generate stream centre lines for analysis of reach planform.

The Physical Form theme of the SRA uses two measures of planform geometry: the mean meander wavelength and sinuosity (channel length divided by valley distance). The variability of river meander bends both in their spacing and form (Figure 1) make them particularly difficult to identify using an automated procedure. Based on the "theory of minimum variance", Langbein *et al.* (1966) proposed that the direction angle of a meandering river channel follows a sine function with distance. They successfully tested this theory in alluvial and bedrock rivers including the Colorado River canyon. Williams (1986) further tested this proposition successfully although restricting their study sites to alluvial rivers.

This paper extends the method of Langbein *et al.* (1966) for the purpose of estimation of meander length without any manual processing of data, such as locating meander bends or identifying valley alignment. It is based upon Langbein *et al.* (1966) and produces a meander length and amplitude over a reach that minimises the least-squares error. Unlike the method of Langbein *et al.* (1966), this method is also objective and reproducible. This paper presents and discusses the method and algorithm with results from implementation in the Murray-Darling Basin. In addition to the method herein, numerous other methods, all based upon Langbein et al. (1966), were investigated. Of all the methods investigated, that presented here produced meander estimates most consistent with qualitative meander estimates and was the most numerically stable, that is with respect to calibration to a global minimum and of minimal parameter covariance.

Figure 1: Six 1 km stream reaches in the Murray-Darling Basin showing variability in meander planform

2. METHODOLOGY

Langbein *et al.* (1966) proposes that the angle between the flow direction at a given point changes along the regional stream flow path to produce a sinusoidal relationship with distance. Thus the wavelength can be estimated by numerically fitting a sine function to this relationship. However, a challenge in applying this method is in defining the regional stream flow path to which the observed angle is calculated. If this method is applied to only a single reach, the regional flow path could simply be located manually. Automated implementation does however require numerical estimation of this regional flow path.

To achieve this, and after trialling a considerable number of alternatives, an algorithm was developed to rotate the reach, fit a *k* order polynomial to estimate the regional flow path, calculate the residuals to this path and then fit the Langbein *et al.* (1966) sinusoidal model to the residuals. The fitting and rotation was undertaken by multi-start Trust-Region non-linear least-squares regression. Below, the developed algorithm is detailed:

1. Rotate the reach coordinates to maximise the x-axis extent:
   i. Calculate the angle from the first to last reach coordinate, where x [L] and y [L] are column vectors of the easting and northing coordinates and n is the number of reach data points:

$$\theta = arctan\left(\frac{y_n - y_1}{x_n - y_1}\right) \tag{1}$$

   ii. Rotate the reach coordinates clockwise by Θ, where **x** and **y** denote vectors of coordinates and T denotes the transposition of the column vector:

$$\begin{bmatrix}\mathbf{x}^T \\ \mathbf{y}^T\end{bmatrix} = \begin{bmatrix}cos(\pi-\theta) & -sin(\pi-\theta) \\ sin(\pi-\theta) & cos(\pi-\theta)\end{bmatrix}\begin{bmatrix}\mathbf{x}^T \\ \mathbf{y}^T\end{bmatrix} \tag{2}$$

2. Calculate the reach chainage, c [L], to point i;

$$c_i = \sum_{j=1}^{i}\sqrt{\left(x_j - x_{j\text{-}1}\right)^2 + \left(y_j - y_{j\text{-}1}\right)^2} \tag{3}$$

3. Calculate the observed reach meander angle, φ, at point i:

$$\varphi_i = arcsin\left(\frac{y_i - y_{i-1}}{c_i - c_{i-1}}\right) \tag{4}$$

4. Estimate the regional flow contribution to the observed reach meander angle, φ, by fitting a polynomial of order k by non-linear least-squares regression.
5. Estimate the residual meander angle, $\varphi_{\mathbf{R}}$, to that estimated by the polynomial.
6. Fit an analytical sinusoidal function to $\varphi_{\mathbf{R}}$ by multi-start Trust-Region non-linear least-squares regression where:
   i. The analytical function to be fit is as follows, where $\varphi_{\mathbf{Rmod}}$ is the modelled meander angle; ω [radians] is a parameter for the maximum meander angle; M [L] is a parameter for the meander wavelength (as measures by the reach chainage); and Δ [radians] is a parameter for the phase shift in the sinusoidal function.

$$\varphi_{R_{MOD}} = \omega sin\left(2\pi\frac{c}{M} + \Delta\right) \tag{5}$$

   ii. And the least squares objective function to be minimised is:

$$\chi^2 = \varphi_{R_{MOD}}^{T}\varphi_{R_{MOD}} \tag{6}$$

Finally, the least-squares calibration of the analytical meander equation (Eq.(5)) required careful implementation to ensure: (i) the parameter estimates were plausible; (ii) the convergence was to the global, and a not local optima; and (iii) multiple shapes of the regional flow were considered. To enforce the estimation of plausible parameters, firstly the Trust-Region gradient Newtonian solver was adopted as it allows inclusion of parameter bounds. There bounds were set as follows:

- $0 \leq M \leq 2000$ metres, where the upper limit was chosen to limit the meander length to double the reach chainage;
- $-2.2 \leq \omega \leq 2.2$ radians, where Langbein *et al.* (1966) notes $|2.2|$ as the maximum angle;
- $-\pi \leq \Delta \leq \pi$ radians, where $|\pi|$ was set to limit a phase shift by half the angular phase.

To increase the probability that the solver converged to a global minimum, the solver was implemented from 400 different initial values for the three parameters. The initial start resulting in the maximum coefficient of efficiency (Nash *et al.* 1970) was selected as the optimal solution for the given order polynomial model (see step 4 above). Finally, to consider multiple regional flow shapes, the entire calibration was then repeated for models with also a parabolic and cubic fitted to step 4 above. The polynomial resulting in the maximum coefficient of efficiency was then selected as the final model, and thus the estimate of meander length.

Lastly, once the optimal model was selected, linear parameter confidence intervals and p-values were calculated. These were derived for a confidence of 0.975.

3. RESULTS

3.1. A Single Reach

To illustrate the method, and to detail some deficiencies, herein the results for reach ID 73010 are presented. Figure 2 presents the observed meander, which is oriented in a north-east direction and shifted to the axis origin. Figure 2 also illustrates the first step of the algorithm, whereby the reach coordinates were rotated to maximise the x-axis extent.

Figure 3 details the remaining two major steps of the algorithm and the final model error. Figure 3a shows the estimated meander angle and a cubic polynomial fit to it. Figure 3b shows the residual angle to the polynomial. Importantly, the first order non-stationarity of Figure 3a has been removed. Figure 3b also shows the sinusoidal function (Eq.(5)) fit to the residual meander angle. This model estimated the meander length as 267.3 ± 3.6 metres and the peak meander angle as 0.8 ± 0.06 radians. The coefficient of efficiency was estimated as 0.742. Finally, Figure 3c shows the error between the sinusoidal function and the estimated residual angle. Importantly, it shows the errors to be serially correlated with chainage. This indicates that the model and parameters are somewhat inefficient as not all of the information from the observed data has been utilised by the model.

3.2. Analysis of All Modeled Reaches

This method has been applied to 1456 river reaches randomly distributed the Murray-Darling Basin, each with a channel length of 1 km. A 1 km reach length was chosen as a compromise between good sampling support and sample size (i.e. reach length versus the number of reaches respectively), while constrained by the available resources for the survey and subsequent data processing. A 1 km reach length was also consistent with the reach length used in other river assessments for example, for environmental flow planning. The mean meander wavelength estimated for these reaches was 560 m and standard deviation across the sites was 260 m. The relative error (97.25% confidence interval divided by estimate) for 95% of the sites was less than 10% (Figure 4).

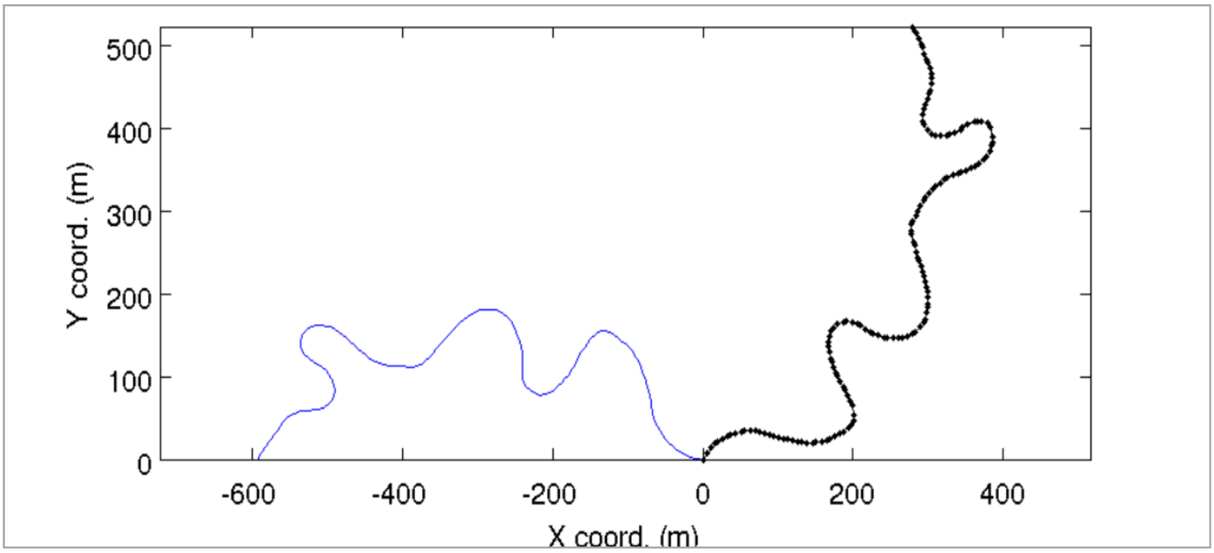
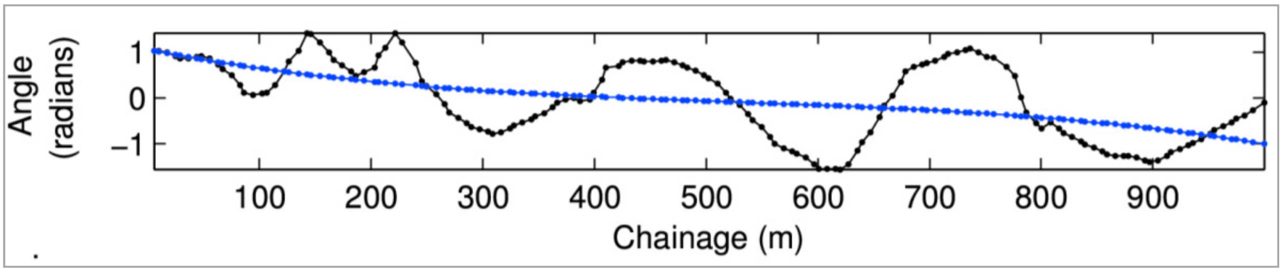
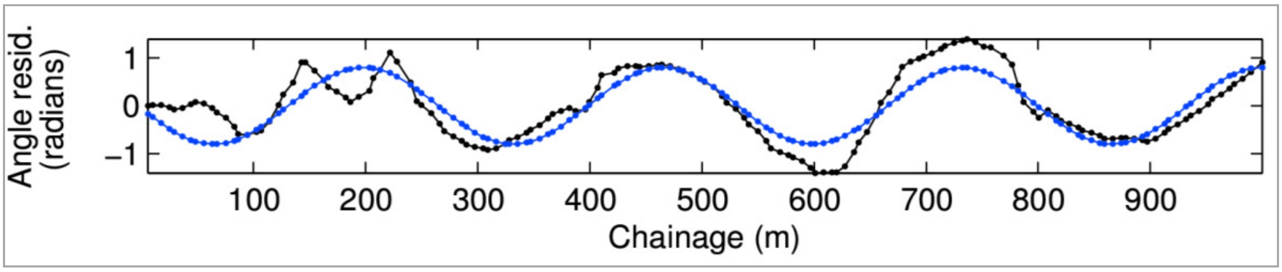


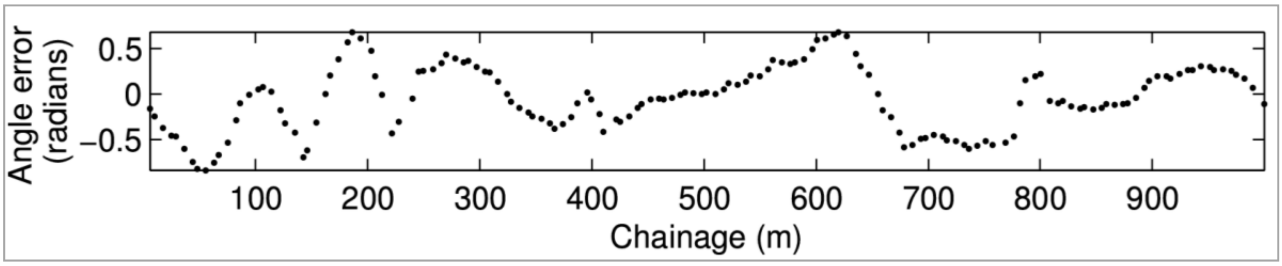
Figure 2. Reach 73010 original coordinate data (black) and rotated coordinate data (blue)



a) Estimated meander angle (black) and the fitting cubic polynomial (blue).



b) Estimated residual meander angle to the polynomial line of Fig. 3a (black) and the fitted sinusoidal model (blue).



c) Error between modelled and estimated meander angle of (b).

Figure 3. Reach 73010 model components and model error.

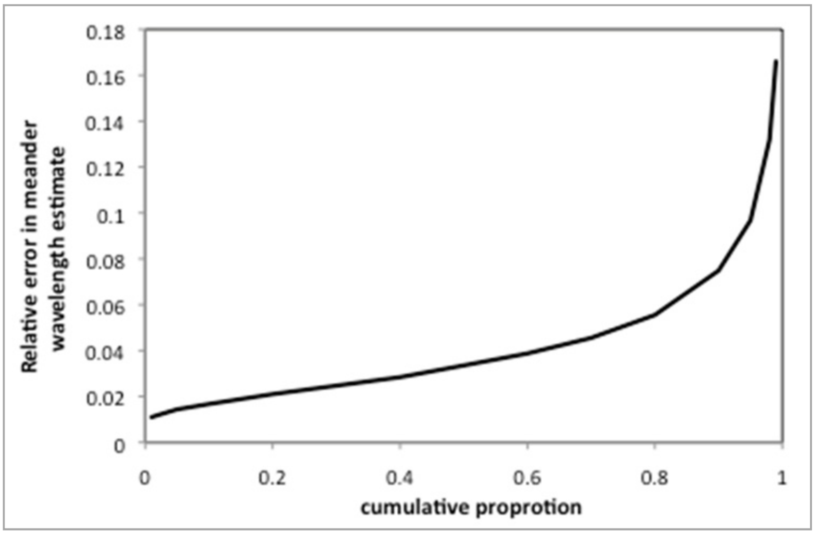


Figure 4: Cumulative distribution of relative error in meander wavelength estimate

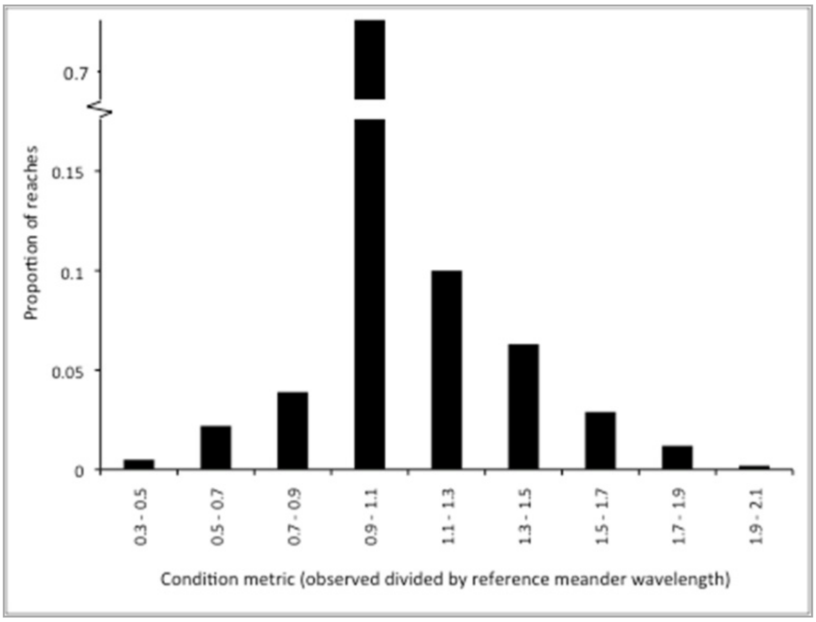


Figure 5: Distribution of meander wavelength condition metric for reaches across the Murray-Darling Basin

For each of these sites we have a reference value of the meander wavelength which is the value expected in the absence of human disturbances since European settlement. The reference values were estimated using a statistical model fitted using data for the 1456 sites. We used a Boosted Regression Tree (BRT) model (Elith *et al.*, 2008) relating meander wavelength to attributes of the river landscape including catchment area, valley slope, flow parameters and measures of human disturbance to catchments. The BRT model explains 36% of the deviance in meander wavelength (equivalent to r^2) of which 30% (i.e. 11% of overall deviance) is explained by catchment disturbances. These model performance results are based on a cross-validation procedure. A "reference range" of meander wavelength was estimated for each site using this BRT model and setting the catchment disturbance metrics equal to zero with upper and lower bounds based on the model error established in the cross-validation stage of model fitting. Cross-prediction was used in this case to ensure model fitting is independent of the observed meander wavelength for all sites. If the observed meander wavelength lies outside these modeled "reference range", then it is considered different from reference. In these cases, the ratio of the observed divided by the expected meander wavelength (using either the upper or lower bound of the expected range which ever is closer to the observed value) was used as a measure of change in meander planform from the pre-European state. A metric value of 1 indicates no change from pre-European, a value greater than 1 indicates an increase in meander wavelength and vice versa for a value less than 1. 72% of the sites lie within the expected range (Figure 5). The observed meander wavelength falls below and above this range at 9% and 29% of sites respectively. These results suggest that the dominant change in river planform from the pre-European state has been an increase in meander wavelength.

4. DISCUSSION AND CONCLUSIONS

Overall, the algorithm developed herein provides a robust, objective and reproducible means for estimation of meander length. Furthermore, those reaches for which the algorithm does not perform satisfactorily can easily be identified by the low coefficient of efficiency.

The algorithm does, however, have some weakness. These must be considered when utilising the results and comprise of the following:

1. The parameter confidence intervals were derived from the assumption that the calibration response surface is locally linear about the global optima. This validity of this assumption has not been assessed, and if incorrect, may result in an under-estimate of the uncertainty.
2. The parameter confidence intervals are likely to be an underestimate of the uncertainty as their calculation ignored the uncertainty in both the coordinate rotation angle and the fit of the linear, parabolic or cubic polynomials.
3. The predictive performance of the algorithm has not been assessed, as it could be undertaken by split a sample calibration-evaluation scheme.
4. The modelled meander length is a single best estimate, in a least-squares sense, to the entire 1 km reach. If a smaller or longer reach chainage were investigated it is likely that the estimate would differ. Therefore, the estimated meander length for the 1km reaches is very likely to be a function of the scale of the application.
5. Highly linear reaches may produce spurious meander length estimates. This will be reflected in a very low coefficient of efficiency. Of the supplied reach data, this occurred for only two reaches.

The most challenging of the above weakness is the reach length. As part of the methodological development for this paper, unsuccessful attempts were made to estimate smaller scale meanders by inclusion of two analytical meander functions into the objective function. A more viable option for future research may be to investigate reaches significantly longer than 1 km and randomly sample from it sub-reaches of varying length and position. Fitting analytical meander functions to these sub-reaches may indicate a reach length for optimal fit of the analytical meander function at each location, and thus giving a more local estimate of meander length.

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