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Does New Zealand visitors follow the Joseph Effect? Some empirical evidence

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Abstract: The report departs from conventional time series analysis and investigates the existence of long memory (LRD) in the stream of daily visitors, arriving from various sources to New Zealand from 1997 to 2010, using selected estimators of the Hurst-exponent. The daily arrivals of visitors are treated as a stream of "digital signals" with the inherent noise. After minimizing the noise (i.e. the presence of short-term trends, periodicities, and cycles) we found the existence of significant long memory embedded in our data of daily visitors from all sources and in the aggregate. Strong evidence of embedded "long memory" implies that Joseph Effect – that good times beget good times and bad times beget bad – whose existence in the underlying process may have interesting implications for tourism policy makers. Our findings suggest evidence of such long term memory in tourist arrival data. Further, unless this long memory effect is taken into consideration, any traditional statistical analysis based on Gaussian and Poisson assumptions may be overly biased.

Keywords: *International tourism, international tourists arrival, long range dependencies, Hurst Index.*

1. INTRODUCTION

Some excellent work has been done on patterns of tourist arrivals using methods such as ARCH, GARCH, GJR and HAR some of the more recent of which are Chang and McAleer(2009), Divino and McAleer(2009, 2010), and Chang, C. L., M. McAleer(2010). All of these involved examining time series, using conventional econometric methodologies. Generally all of the methodologies used in these papers have used sophisticated statistical tests (along with their associated assumptions) to ensure the efficacy of their conclusions, especially, in dealing with embedded long range dependencies. Conventionally, volatility clustering can be examined directly using methodolgies mentioned above and LRD can be captured by using such methods as HAR proposed by Corsi (2004). This paper proposed an alternative approach to detect long range dependencies (LRD) used by digital signal processing discipline. Here a time-series is treated as a stream of signal to be analysed directly in an attempt to quantitatively measure relative LRD, if any, of different signal streams. Note that here the signal's LRD behaviour is elicited from the level of signal amplitude and not necessarily from the higher statistical moments of the signal. One common way of quantifying LRD is the *Hurst Index or Hurst exponent (H)* where the presence of LRD is inferred at certain level of H threshold.

Much has been done in areas of computer science (Pacheco. Roman and Vargas, 2005), economics and finance (Taqqu, Teverovsky and Willinger, 1999), and in the physical sciences using various *Hurst Index (H)* estimators. However, it is well known that LRD estimators are themselves unstable and erratic (Karagiannis et al, 2006). Further, H cannot be calculated in any definitive or direct way and has to be estimated as a by-product of some statistical estimating procedures. Depending on the estimator used, if sufficient care is not taken to ameliorate the underlying limitations of each of these estimators, the results can sometimes be mutually contradictory and conflicting (Karagiannis, Faloutsos and Reidi, 2002). For deeper methodological issues one should consult Beran (1992) or Allan (1996)

Some of the main problems in estimating the H-index are the presence of short term trends, non-stationarity, periodicity and noise. Regardless, it is better to test whether LRD and other anomalies are present in a time series before deciding whether normal Gaussian analyses are appropriate rather than just assuming that a time series is distributionally Gaussian or Poisson. The statistical properties of a series with LRD can be quite different from those of a series that are iid. For instance, the variability properties of sample means of assumed iid observations are far from valid in the presence of LRD. In the area of tourism study, as in all study of involving non-Gaussian time series, the presence of LRD is of practical significance; the least of which is that the presence of undetected or un-ameliorated LRD may pose potentially significant problems in the statistical and substantive conclusions derived. Further, the presence of LRD could be interpreted as "long memory" whose presence implies "persistence" which gave rise to the what Madelbrot et al (1968) called the "Noah" and "Joseph" effects, whose manifestations tend to lead to an analog of higher moments (volatility clustering) examined in Chang and McAleer(2009), Divino and McAleer(2009, 2010), and Chang, C. L., M. McAleer(2010)

Theoretically, there are several ways of defining long memory process ("long range dependence" or LRD). An intuitively popular definition is couched in terms of the auto-covariance function, $\rho(k)$, such that a long memory process is present if in the limit, $k \to \infty$:

$$\rho(k) \sim k^{-\alpha} L(k)$$

where $0 < \alpha < 1$and $L(x)$ is a slow varying function. Hence a stationary process X_t is long-range dependent, if there exists a real number $a \in (0,1)$ and a constant $c_1 > 0$ such that

$$\lim_{k \to \infty} \rho(k) / [c_1 k^{-a}] = 1$$

Where $\rho(k)$ represents the sample correlation function and k is number of lags . The definition states that the autocorrelation function of long memory processes, decay to zero with rate approximately k^{-a}.-The parameter that characterizes long-range dependence (LRD, "long memory") is the Hurst exponent (H), where $H = 1- \alpha/2$. Long-memory occurs when $½ < H < 1$ ("persistence") and $0 < H < ½$ ("anti-persistent"). Long memory process can generate non-periodical cyclical patterns as ones observed by Hurst (1951) for the Nile River, where long periods of drought are followed by long periods of plenty. Mandelbrot and Wallis (1968) called this phenomenon as the "Joseph" or "Hurst" effect.

3. OBJECTIVES AND METHODOLOGY

One of the main objectives is to assess whether there are pronounced LRD presence in these series, if so, there could be profound implications for normal statistical analysis of these series. Further, if LRD is strong (H>>0.5), after eliminating the presence of known "contaminants" such as short-term trends, non-stationarity, periodicity and noise, one might infer that there is LRD along with all the implications of the manifestations of persistence, self-similarity and the presence of an underlying fractal structure. All these are indications of the presence of "heavy tails" in the distributions of the time series concerned. Presence of a high H-exponent suggests "persistence" and the existence of the "Joseph Effect" where good years beget further good years and bad begets further bad. This could provide important signposts for long term policy making, especially in the strategic dimensions of tourism infra-structural development, market development and policy making.

The following estimators used here are chosen because they are the most common estimators. Their efficacies and limitations are well documented. These H estimators are applied to several data series of daily tourist arrivals procured from the New Zealand Statistics Department for the period spanning 1 September 1997 through 31 October 2010, totalling close 5000 data points. These comprise daily arrivals from Australia, China, Japan, UK, USA and the rest of the world.

3.1 Time Domain Estimators

Rescaled Range (R/S) Estimator
The R/S estimator is a statistical estimator of H such that:

$$E\left[\frac{R(\tau)}{S(\tau)}\right] = Cn^H$$

where $R(\tau)$ is the amplitude range over a time window, τ, scaled to the standard deviation, $S(\tau)$, of the range. Here, $R(\tau) = \max\big(X(t,\tau)\big) - \min(X(t,\tau))\ for\ 1 \le t \le \tau$ and $S(\tau) = \sqrt{\frac{1}{\tau}\sum_{t=1}^{\tau}(\xi(t) - \langle\xi\rangle_\tau)^2}$ where $X(t,\tau) = \sum_{u=1}^{t}\xi(u) - \langle\xi\rangle_\tau$ and $\langle\xi\rangle_\tau = \frac{1}{\tau}\sum_{u=1}^{\tau}\xi(t)$. The R/S for any given τ is R/S(τ) = $\frac{R(\tau)}{S(\tau)}$, and H is the regression slope of log τ against log R/S(τ).

Absolute moment Estimator (A/M)
Here, a time series, X_t, is divided into blocks of size *m*, such that:

$_{(k)}X^m = \frac{1}{m}\sum_{i=(k-1)m+1}^{km} X_i$ where *k = 1, 2,... N/m for a series* X_i, *i =1, 2,.... ,N* and *k* is the index that labels the block. The sum of the absolute values of the series is computed for various m, i.e.

$$A/M^{(m)} = \frac{1}{N/m}\sum_{k=1}^{N/m}\left|X_k^{(m)} - \bar{X}\right|,$$

where $\bar{X}$ denotes the original series' sample mean. Regressing the log of this statistic against the log of *m* should provide a line with a slope of *H-1,* where *H* is the Hurst exponent.

Variance Method Estimator (VM)
VM exploits a characteristic property of the variance inherent in LRD processes that the variance of the sample mean converges to zero slower that the 1/N where N is the sample size. If LRD is present, then we have, for large N:$Var(\bar{X}_N) \sim cN^{2H-2}$, where c > 0 and $\bar{X}_N$ is the sample mean. If we divide a time series, X_t, into block size of m, and within each block, m, aggregate the sub-series to produce a new sub-series, $X^{(m)}$ such that $_{(k)}X^{(m)} = \frac{1}{m}\sum_{i=(k-1)m+1}^{km} X_i$ where k = 1, 2,.... and k is the label index of the block. The sample variance of $X^{(m)}$, is calculated as

$$s^2(m) = \frac{1}{\left(\frac{N}{m}\right) - 1}\sum_{k=1}^{N/m}\left({X_{(k)}}^{(m)} - \bar{X}\right)^2$$

Where $\bar{X}$ denotes the global mean. Regressing log $s^2(m)$ against log(m) for each m, successively, we should have a line with a slope of 2H-2, from which H can be inferred.

Variance Of Residuals Estimator
A time series, say, {X_i, i ∈ positive integers}, is divided into blocks *m*, defined as Ψ_m= { Ψ_1^m, Ψ_2^m, …. Ψ_n^m, . . }, a derived new series from {X_i} with blocks size *m*. Each $\Psi_i^m \stackrel{\text{def}}{=} \{X_{(i-1)m+1}, X_{(i-1)m+2}, \dots, X_{im}\}$ represents a block of size *m* of the original series. To every Ψ_m, the partial sum series $P(\Psi_i^m) = \{P_i^m(1), P_i^m(2), \dots, P_i^m(m), \dots\}$, where each of $P_j^m(j) = \sum_{j-1}^{m} X_{(i-1)m+j}$ and $P^m = \{P(\Psi_1^m), P(\Psi_2^m), \dots, P(\Psi_i^m), \dots\}$, can be computed. Then a least square line is fitted to the partial sum series P^m to give a new series $\mathcal{Z}^m = \{Z_1^m, Z_2^m, \dots, Z_i^m, \dots\}$ ("the least square series"). The variance of the residual is given as:

$$V_{res}^{m}(i) = \frac{1}{m}\sum_{j=1}^{m}(P_i^m(j) - Z_i)^2$$

If the process is known to be LRD, then the median of the residual series behaves as $Med(V_{res}^{m}) \sim m^{2H}$ for large m. A log-log plot of $Med(V_{res}^{m})$ against varying m should give a straight line with slope 2H, where H is the Hurst exponent.

3.2 Frequency Domain Estimators

Periodogram Estimator
The periodogram may be defined as follow:

$$S(v) = \frac{1}{2\pi N}\left|\sum_{j=1}^{N} X(j)e^{ijv}\right|^2$$

where v is the frequency and X is a given time series of length N. Given a series of finite variance, the $S(v)$ is an estimator of the spectral density of X, then a series with LRD will have a spectral density $S(v)$ proportional to $|v|^{1-2H}$at the lower frequencies close to zero. Hence, a log-log plot of the $S(v)$, also known as the *periodogram*, against *v, frequencies*, should present a straight line with a slope of 1-2H. The frequencies used to estimate H are best taken at the lowest 10% of the spectrum in order to comply with the requirements stated above.

Local Whittle Estimator
It was stated in Taqqu and Teverovsky (1997) that the local Whittle estimator is semi-parametric and assumes the existence of LRD. :Local Whittle ("LWhittle") is preferred here because it makes less *a priori* assumptions that the standard Whittle. Since Local Whittle is based on the periodogram stated above, it's focus is also centred around low frequencies of the spectrum. LWhittle differs from the periodogram approach by its adding an extra parameter, M, which is an integer of less than N/2, and satisfying (1/M) + (M/N) → 0 as N→∞. Assuming only the functional form of spectral density we have: $f(v) \sim G(H)|v|^{1-2H} as\ v \rightarrow 0$

The objective is to minimize:

$$R(H) = \log\left(\frac{1}{M}\sum_{j=1}^{M}\frac{S(v_j)}{v_j^{1-2H}}\right) - (2H-1)\frac{1}{M}\sum_{j=1}^{M} log v_j$$

Please note that unless the series is understood to be ideal, then M should be as small as possible, which is in effect using frequencies close to zero to minimize the effect of short range effects on the spectral density.

Abry-Veitch Estimator
Generally, the Hurst exponent is derived from a wavelet transform (in in case, Daubechies wavelets) of the time series X = (x_1, x_2, ... x_n). Given a series with long memory stochastic process, the variance at level *i* of the wavelet coefficients, $d_x(i,j)$ is given by:

$$Var(d_x(i,..)) = \frac{\sigma^2}{2}V_\psi(H)(2^j)^{2H+1}$$

where $V_\psi(H)$ depends on a particular wavelet chosen and the Hurst exponent. V is defined by $V_\Psi(H) = -\int_{-\infty}^{\infty}\gamma_\psi(\tau)|\tau|^{2H}d\tau$
Taking the log of the variance of wavelet coefficients above we have $logVar(d_x(i,..)) = (2H+1)j + K$
where K is a constant.

Specifically, a time average μ_i, of $d_x(i.j)$ is computed at a given scale. $\mu_i = (n_i)^{-1}\sum_j^{n_i} d_x^2(i.j)$, where n_i is the wavelet coefficient number at scale *i* and *n* the time series points. The Hurst exponent is estimated from the slope of a linear regression model stated as follows:

$$log_2(\mu_i) = log_2\left(\frac{1}{n_i}\sum_{j=1}^{n_i} d_x^2(i,j)\right)$$

where i = 1, 2, ... ($log_2(n)$.

4.0 DATA AND PROCEDURES

4.1 Total Tourist Arrivals from all sources
A plot of the Total Arrivals from all sources is shown in Figure 1. Total Daily Arrivals: 1-9-1997 to 31-10-2010. The plot is typical of tourist arrival patterns and conforms to patterns in other studies (Medeiros C, McAleer M, Slottje D, Ramos V and Rey-Maquieira J, 2008), with distinct peaks and troughs of a mix of periodicities, the bane of most LRD estimators. Further, the existence of trend and noise do not help matter much either.

The Autocorrelation Function plot shown in Figure 2 shows the effects of periodicities. These are indications that caution should be exercised where Gaussian assumptions are necessary. For our purpose, we need to rid the series of these impediments as much as possible. Karagiannis, et al. 2006 suggests *Randomized Buckets* for ridding the series of most of these impediments. The procedure generally involves taking selected sub-series of a selected time series, randomizing these to "control the amount of correlation at different time scales." The procedure comprises External Randomization, Internal Randomization and Two Level Randomization. In this paper we will restrict ourselves to Internal Randomization.

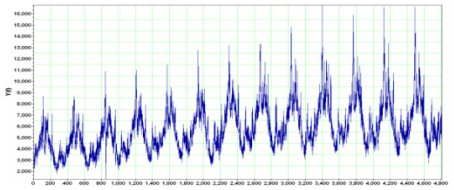

Figure 1. Total Daily Arrivals: 1-9-1997 to 31-10-2010

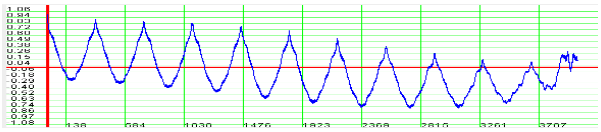

Figure 2. Autocorrelation Function of Total Daily Arrivals (Raw)

Briefly, the series is divided into segments ("buckets"), where intra-bucket data elements within the buckets are randomized, without the order of each of the contiguous buckets being changed. This has the effect of minimizing any correlations between intra-bucket data elements while correlations among the buckets themselves are maintained. If the original series contains LRD, then the ACF after this procedure should still manifest a "power-law" structure as the existence of LRD should be preserved.

The Internal Randomizing ("IR") procedure is applied to the Total Arrival data series and the result can be seen in Figure 3. Although the periodicity has been dampened, it is still discernible as is evident in the ACF presented in Figure 4. ACF of total Tourist Arrivals After Internal Randomization (Bucket size =100) The IR procedure is now repeated, by increasing the bucket size, until all discernible evidence of periodicity or short-term trends disappears.

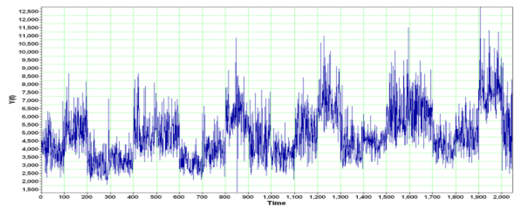

Figure 3. Total Tourist Arrivals after Internal Randomizing (Bucket size=100)

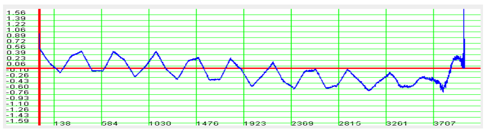

Figure 4. ACF of total Tourist Arrivals After Internal Randomization (Bucket size =100)

We found that at bucket size of approximately 300 , Figure 5 manifests with its corresponding ACF in Figure 6. Although the ACF here is not parabolic, it does show the characteristic slow attenuation of the function. We seem to have rid the data-set of most periodicity and short term trends.

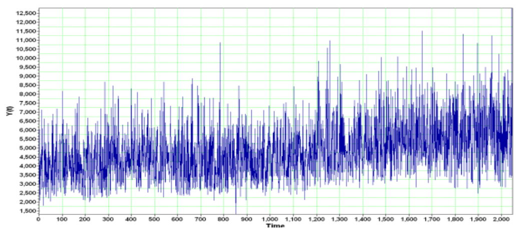

Figure 5. Total Tourist Arrival after Internal Randomization (Bucket size = 300)

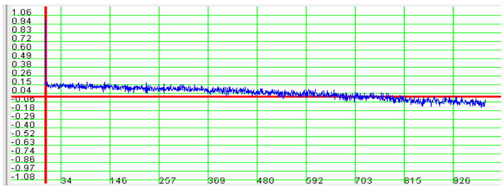

Figure 6. ACF of Total Arrivals Post IR (Bucket size = 300).

From the "cleansed" dataset, we estimate the Hurst Exponents ("H") using the selected H estimators derived from two publicly available software, namely, SelQos and SELFIS. First, the following time-domain H estimators from the software were applied: i.e. Rescaled Range (R/S), Absolute Momentum (A/M), Variance Method (VM), Variance of Residuals (VoR) and Modified Allan Variance (MAV). Then, the following frequency domain estimators were respectively applied: i.e. Periodogram (PDM), Local Whittle (Lwhittle) and Abry-Veitch Method (AV). Each of these estimators has their own strength and weaknesses even after having rid the data series of short term "contaminants". Further, note that both SelQoS and SELFIS implement these estimators differently Examining the outputs from both the SelQoS and SELFIS implementations and combining the all the H estimates (i.e. time and frequency domains) we have a Combined Median of 0.627 and Combined Mean of 0.683. The SELFIS implementation of the estimators was criticized in Pacheco et. al (2005) and Pacheco and Torres-Roman (2006) for consistently underestimating the H exponent each estimators under various controlled conditions. Hence we prefer SelQoS and used its implementations.

4.2 Tourist Arrivals from Australia, China, Japan, UK, USA, Rest of the world

Applying similar procedures outlined in the last section to various sub-components of the Total Arrivals, between 1/9/1997 and 30/10/2010, we have the estimated the following H-exponents after attempting to eliminate all signs of short-term trend, periodicities and cycles using the appropriate internal randomization bucket size (i.e. "critical" bucket size). The "critical" bucket size is the approximate bucket size used in the IR process when all the short-term trends, periodicities and cycles are believed to have been eliminated in the IR process. In this case, all data sets seemed to reach this "critical" level at a bucket size of about 300. The transformed set of data is then used to estimate H using the various H-estimators provided by SelQoS. The results are summarized in Table **Error! Reference source not found.**1.

Source	Time Domain						Frequency Domain					Combined Time-Frequency Domains		
Country	R/S	A/M	VM	MAV	Median	Mean	PDM	Lwhittle	AV	Median	Mean	Median	Mean	Std-Dev
Australia	0.8325	0.7645	0.7207	0.5307	0.7426	0.7121	0.6880	0.8483	0.5331	0.6880	0.6898	0.7207	0.7025	0.1296
China	1.0350	0.9654	0.9284	0.7522	0.9284	0.8820	0.7643	0.9963	0.4936	0.7643	0.7514	0.8464	0.8167	0.1888
Japan	0.6526	0.5581	0.5461	0.5970	0.5775	0.5884	0.6351	0.5954	0.5151	0.5954	0.5819	0.5954	0.5856	0.0491
UK	0.8182	0.7446	0.7221	0.5722	0.7334	0.7143	0.5641	0.7323	0.4678	0.5641	0.5881	0.7221	0.6602	0.1259
USA	0.7980	0.9408	0.9102	1.2884	0.9102	0.8830	0.8280	1.2968	0.8472	0.8376	0.8376	0.8472	0.8648	0.0591
RoW	0.8504	0.7778	0.7610	0.5599	0.7694	0.7373	0.5909	0.6504	0.5297	0.5909	0.5903	0.6504	0.6743	0.1230
Aggregated	0.8701	0.8153	0.7865	0.5681	0.8009	0.7600	0.6541	0.8146	0.5195	0.6541	0.6627	0.7865	0.6627	0.1477

Table 1. A Summary of Time and Frequency Domain Estimates of H-exponents

5. CONCLUSIONS/IMPLICATIONS

It can be discerned that arrivals from Australia, China, UK and USA shows very high mean and median H-indices for both time and frequency estimators giving a strong indication of LRD and the Joseph effects. Arrivals from Japan and the Rest of the World mixed mean and median H-indices, but all still exceeds 0.5, indicating the presence of moderate LRD.
As is evident from the strengths of all the estimates of H-exponents, one may infer the following from the results presented:

1. In attempting to elicit information from any set of data of this genre, one has to be extremely careful when using statistics that rely on Gaussian and Poisson assumptions for the purpose of forecasting even after performing most of the required standard tests. Pre-processing of the data set to determine the existence of LRD may be necessary in order to take into account the effects of LRD on one's statistical conclusions.

2. One may infer that Joseph Effect may be in operation if one finds LRD in a reasonably "sanitized" data set, i.e. a data set that's rid of short term trend, cycles and other periodicities. Internal Bucket shuffling is one of the methods of minimizing such impediments. Our findings support the existence of LRD hence the Joseph Effect.

3. At the moment although one may infer the existence of Joseph Effect, we are unable to determine the duration of such effect. This is subject to ongoing research.

REFERENCES

Allan, D.W.(1966). Statistics of Atomic Frequency Standard, *Proceedings of the IEEE*, 54, No. 2, 221-231

Beran, J. (1992). Statistical Methods for Data with Long –Range Dependence, *Statistical Science*, vol. 7, pp 404-16.

Chang, C. L., and M. McAleer (2009). Daily tourist arrivals, exchange rates and volatility for Korea and Taiwan, *Korean Economic Review,* 25, 241-267.

Chang, C.L., and M. McAleer (2010). Aggreation, Heterogeneous Autoregression and Volatility of Daily International Tourist Arrivals and Exchange Rates, *http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1504651*

Coleman, R. (2009) Research Article, Long Memory of Pathfinding Aesthetics, *International Journal of Computer Games Technology*, Vol. 2009, Article ID318505, page 3.

Corsi, F. (2004). A simple long memory model of realized volatility, *http://www.core.ucl.ac.be/~laurent/SL_fichiers/LSE/pdf/A%20Simple%20Long%20Memory%20Model%20of%20Realized%20Volatility.pdf*

Divino, J.A. and M. McAleer (2009). Modelling and forecasting sustainable international tourism demand for the Brazilian Amazon, *Environmental Modelling and Software,* 24, 1411-1419

Divino, J.A. and M. McAleer (2010). Modelling the growth and volatility in daily international mass tourism to Peru, pending, *Tourism Management.*

Geweke, J. and S. Porter-Hudak (1983). The estimation and application of long memory time series models, *Journal of Time Series Analysis*, 4, 221–238

Hurst, H., (1951). Long term storage capacity of reservoirs, *Transactions of the American Society of Civil Engineers*, 116: 770-799

Karagiannis T and M. Faloutsos (2002). SELFIS: A Tool For SelfSimilarityand LongRange Dependence Analysis. Software retrieved from Microsoft Research on 21 July 2010 from: *http://research.microsoft.com/apps/pubs/default.aspx?id=71485*

Karagiannis T., M. Molle, M Faloutsos (2006). Understanding the Limitations of Estimation Methods for Long-Range Dependence, *UCR-CS-2006-10245*, November, 2006.

Karagiannis T., M. Faloutsos, and R. Reidi (2002). Long Range Dependence: Now you seeit, now you don't. *IEEE GLOBECOM*, Global Internet Symposium, 2002.

Kunsh. H. R. (1987) *Statistical aspects of self-similar processes*, 1, 67–74, Utrecht, VNU Science Press.

Mandelbrot, B. (1967). The variation of some other speculative prices, *Journal of Business*, 40, 393-413

Mandelbrot, B. and J. Wallis, (1968). Noah, Joseph and operational hydrology, *Water Resources Research*,4, 909-918.

Mandelbrot, B.(1971) When can a price be arbitraged efficiently? A limit to the validity of the random walk and martingale models, *Review of Economics and Statistics*, 53, 225-236.

Medeiros C, M. McAleer, D. Slottje, V. Ramos and J Rey-Maquieira (2008). An alternative approach to estimating demand: Neural network regression with conditional volatility for high frequency air passenger arrivals, *Journal of Econometrics*, 147 (2008) 372-383.

Pacheco, J C R, D. Roman, and L. Vargas (2005). Retrieved: 1 September 2010, from *http://biblioteca.coqcyt.gob.mx/bvic/Captura/upload/Good-and-efficient-hurst-ANACON.pdf*

Pacheco, J C R, Torres-Román D,, L. Estrada-Vargas (2005). Good and efficient hurst estimates with SELQOS. Software retrieved August 2010 from: *http://biblioteca.coqcyt.gob.mx/bvic/captura/resyaz.php?busqueda=GOOD%20AND%20EFFICIENT%20HURST%20ESTIMATES%20WITH%20SELQOS&where=meta&atributos=2&formato=2&tipo=normal&Zservers[0]=localhost:9999&Cantidad=5*

Pacheco, J C r and Torres-Roman D, 2006. "A Tool for Long-range Dependent Analysis via the R/S statistic", *Proceedings of the 15th International Conference on Computing (CIC'06)*, 2006 IEEE.

Pacheco JCR, D. T. Roman, L. E. Vargas and O.E.R. Ferrera. (2008). A High Performance Tool for Long-Memory and Self-similarity Analysis, retrieved 1 September 2010 from:*http://biblioteca.coqcyt.gob.mx/bvic/Captura/upload/A-HIGH-PERFORMANCE-ANACON.pdf.*

Peters, E.E. (1994). *Fractal Market Analysis*, Wiley, New York

Taqqu, M. S and V.Teverovsky (1996). Semi-parametric graphical estimation of long memory data, in P.M. Robinsion & M. Rosenblatt (Eds.), "*Athens conference on applied probability and time series*" Vol 2, Time series analysis in memory of E.J. Hannan. Springer.

Taqqu, Murad S. And V. Teverovsky (1997). Robustness of Whittle-type estimators for time series with long-range dependence, *Stocahastic Models*, 13: 4, 723-757.

Taqqu, Teverovsky and Willinger (1999). Stock Market prices and long-range dependence, *Finance and Stochastics*, Springer-Verlag, 3, 1-13.

Veitch, D. and P. Abry (1998). A Wavelet Based Joint Estimator of the Parameters of Long-Range Dependence, *IEEE Transactions on Information Theory*, 45

Wallis, J.R. and N. C. Matalas (1970). Small Sample Properties of H and K Estimators of the Hurst Coefficient H, *Water Resources Research*, 6, 6, 1583-1594.