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Simulating Trader Manipulation in a Limit-Order Driven Market

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Abstract: Use of trading strategies to mislead other market participants, commonly termed market manipulation, has been identified as a major problem faced by present day stock markets. Although some mathematical models of market manipulation have been previously developed, this work presents a framework for manipulation in the context of a realistic computational model of a limit-order market.

In this work, the Maslov (2000) limit order market model is extended to introduce a manipulator and technical traders. The Maslov model simplifies the concept of a market place for a particular stock by constructing a limit-order book contents which are manipulated by an infinite pool of uninformed traders. The developed model is considered in terms of market dynamics, overall profit, and detectability of the manipulation strategy. Concepts such as the role of supply and demand in determining price direction, how technical traders extract information from trading, and how manipulators use these principles to alter prices are presented via the model properties.

Uninformed traders in the original Maslov model do not consider behaviour of other market participants when taking trading decisions. Because of this exogenous nature of uninformed traders, manipulation is not possible in the Maslov model. In contrast, technical traders base their trading on the behaviour of informed traders that are believed to be trading in the market. These technical traders utilise a Bayesian learning model to extract information from informed trading and use that information when making their buy/sell decisions. This behaviour of technical traders makes manipulation possible in our model.

The manipulator pretends to be informed and misleads the technical traders in performing a "pump and dump" manipulation scenario. We divided this manipulation strategy into three periods termed, "ignition period", "momentum period", and "call-off period". In the ignition period, the manipulator buys at higher prices (i.e., only market orders) giving an appearance of a higher activity causing the price to go up. This false signal affects technical trader behaviour and their demand further raises the price in the momentum period. The manipulator finishes his manipulation strategy in the call-off period by dumping his shares in a rapid rate causing the price to collapse.

The presence of technical trading adds a persistence behaviour (momentum) to the Maslov prices. The manipulator profit is higher when there are more information seekers (i.e., technical traders) in the market and the presence of manipulation reduces the technical trader profits. We also show that the level of information asymmetry between buying and selling makes the manipulation possible and more profitable. Moreover, when technical traders believe that there exists more informed trading in a market, the manipulator effort required to mislead the market is comparatively low and hence the manipulator profit is higher.

In future, we are planning to find evidence of real stock manipulation characteristics such as high volatility and low market efficiency in our manipulation model. There is also an opportunity to extend our model to study the dynamics of other price manipulation mechanisms such as "marking the close" and "painting the tape". Moreover, measures such as volatility (ARCH/GARCH effects), properties of price return distributions, and market efficiency measures such as the Hurst exponent could be used to detect the manipulation period.

Keywords: Limit order markets, market manipulation, market micro-structure models, bayesian learning

1 INTRODUCTION

A stock market is a mechanism to trade company stocks among market participants at an agreed price. These market participants compete with each other with the expectation of maximising their trading profits. The imbalance between supply and demand of these participants determines the price direction.

Market participants are characterised in terms of the information that they utilise in making a trading decision. Common trader categories are liquidity traders (uninformed), technical traders (information seekers), and informed traders. Liquidity trader actions are exogenous to the market conditions. Informed traders have private information and they utilise that information to take advantage over others. Technical traders generate information using past trading data and are influenced by their perception of informed trading.

A common form of market structure is the use of a double-auction or limit-order mechanism for controlling the bid and ask price offers for a stock. This approach is characterised by a limit-order book that stores current bid and ask prices, and a last traded price representing the last sale (essentially when bid and ask prices are crossed). Market orders are submitted as buy/sell at the current market prices. Maslov (2000); Rosu (2009); Krause (2006) have developed computational models that incorporate the mechanics and trader behaviours of limit order markets.

The limit order market model developed by Maslov (2000) simplifies the concept of a market place for a particular stock by constructing a limit-order book contents of which are manipulated by an infinite pool of uninformed traders. In each time step, a new trader is drawn from this pool as a buyer (probability q_b) or seller (1-q_b), and either a limit-order (probability q_{lo}) or a market order (1-q_{lo}) is set. If a limit-order is set, a value is drawn from a distribution Δ as an offset from the last traded price p(t) (trader computes buy/sell limit order price with a negative/positive Δ from p(t)). Each buy or sell involves a single unit of a stock. Traders are termed liquidity traders since they do not consider the state of the current limit-order book, such as the spread, or any past patterns of the last traded price. All limit orders are assumed to be "good till canceled".

Withanawasam et al. (2010) introduced an alternative pricing method (denoted by M^*), which computes limit buy/sell order prices with a negative/positive offset (Δ) from best ask/bid price of the Maslov order book (i.e., contra side best prices). If the contra side is empty, the limit prices are computed with respect to the last traded price. The M^* method reduces the unrealistic cone shape patterns that are observed in the original Maslov model.

Use of trading strategies with the intention of misleading other market participants is called "market manipulation". One such manipulation scenario is commonly termed "pump and dump", where manipulators pretend to be informed and mislead the technical traders. They buy at successively higher prices, giving the appearance of activity at a higher price than the actual market value, and then sell or dump shares at an inflated price. Chakraborty and Yilmaz (2004) and Mei et al. (2004) analysed this manipulation scenario in a stock market. Allen and Gale (1992) discussed the possibilities of trader based manipulation and showed that a manipulator can pretend to be informed and mislead a market. Allen and Gorton (1992) showed that the asymmetry between the information associated with buying and selling (i.e., a buy contains more information than a sell) leads the manipulator to buy causing a higher effect to the price and sell with a lower effect. Aggarwal and Wu (2006) presented common characteristics of manipulated stocks. Some other notable papers in this area are Berle (1938) and Kyle and Viswanathan (2008).

Although some mathematical models of market manipulation have been previously developed, this paper presents a framework for manipulation in the context of a realistic computational model of a limit-order market. The Maslov model is extended by introducing technical traders and a manipulator. Technical traders base their trading decisions on market information (informed trading), which makes their trading dependent on the actions of other traders in the market. The manipulator induces wrong information to the market to mislead technical traders as an attempt to make illegal profits ("pump and dump").

Technical traders in this model utilise a Bayesian learning technique to generate information from informed trading and use that information in determining their probability of buying/selling. A Bayesian belief tree and observed trading actions (orders and trades) are employed in this Bayesian learning technique. Past work of Glosten and Milgrom (1985), Almgren and Lorenz (2006), Hautsch and Hess (2007), and Pastor and Veronesi (2009) used Bayesian learning in modelling stock markets.

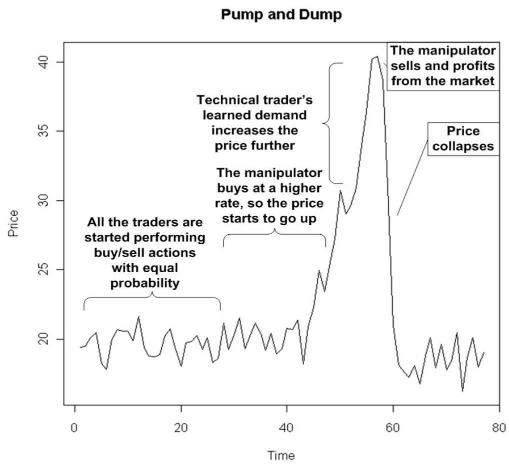

Figure 1. A graphical representation of the behaviour of the model

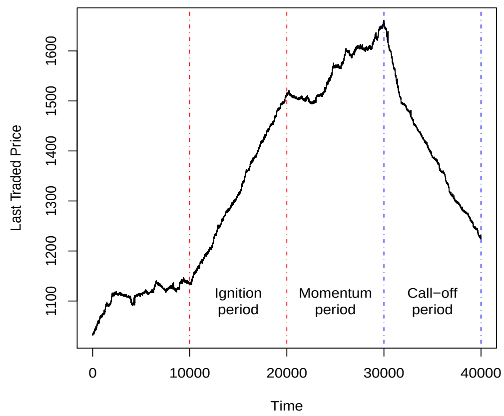

Figure 2. The price impact of the "pump and dump" manipulation

The manipulator uses the price behaviour in response to supply/demand along with the behaviour of technical traders when attempting to inflate or deflate the price and mislead the market. In doing this, the manipulator continues to buy with a higher probability for a certain period of time. This results in an increase of the buying probability of technical traders and influences them to buy more. This artificial demand created by the manipulator inflates the price. After allowing the technical traders to raise the price further, the manipulator repeatedly sells his stocks and profits from the market. The supply created by the manipulator changes the buying probability of technical traders and the price collapses. This behaviour of the model is depicted in Figure 1 and the price impact of the pump and dump manipulation is illustrated in Figure 2.

The developed market manipulation model is presented in Section 2. Results findings, implications of findings, and future work are discussed in Section 3.

2 METHODOLOGY

The "pump and dump" manipulation scenario is modelled through a manipulator who misleads the technical traders by introducing misleading information to the market. In doing that, we extended the M^* model by adding a manipulator and n_T number of technical traders. The number of liquidity traders are also limited to n_L. This limitation was introduced to make the comparison of trading profits of these traders feasible. All these traders are assumed to have initial stocks of size H and money M. The final profit is computed by the difference between initial and final wealth of the trader. In computing the wealth the stocks at hand is liquidated with respect to the last traded price at the moment. Also in nullifying the effect of market parameters such as price, profits of manipulators and technical traders are computed with respect to the average liquidity trader profit.

All the traders (i.e., n_T+n_L+1) are considered to be in a pool and are selected for trading uniformly throughout the simulation. The simulation (i.e., t_T time steps) is divided into two periods: non-manipulation period and manipulation period. In the manipulation period the manipulator uses a different strategy to introduce the manipulation. In the non-manipulation period his strategy is similar to the strategy of uninformed traders. The manipulation period starts after t_S time steps.

Technical traders utilise a Bayesian learning model to generate information from past informed trading. Using this Bayesian learning model, each technical trader models his probability of price going up at

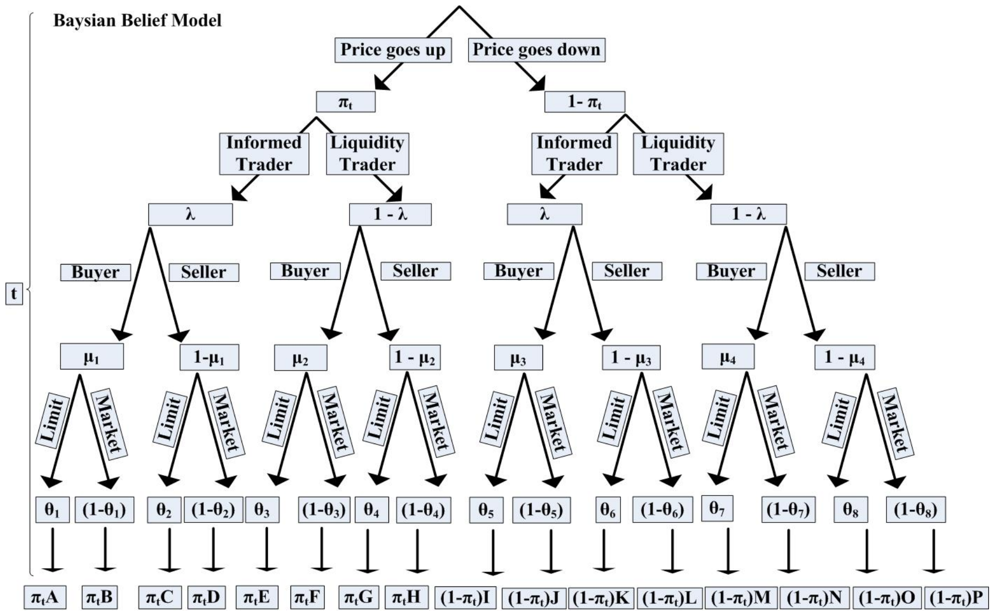

Figure 3. Bayesian diagram to compute posterior probabilities

time t as π_t (starting π_t is π_0). This probability is utilised in determining their probability of buying q_b^T. The observed trading actions (orders/trades) are used in revising π_t. In other words, the posterior probability of π_t given the observed trading action at time t becomes the probability of price going up at time t+1. A Bayes probability tree based on the trader's beliefs (Figure 3) is used in computing the posterior probabilities.

When forming this Bayesian belief model, a technical trader takes the following conditions into consideration: whether the trader involved in the next order/trade is likely to be an informed trader or a liquidity trader, whether this trader is acting as a buyer or a seller, and whether he is placing a limit order or a market order. All these conditions are characterised by conditional probabilities as shown in Figure 3.

Based on this Bayesian learning process, the prior probability of price going up at any time t can be stated as:

$$\pi_t = \frac{\pi_0(A+E)^w(B+F)^x(C+G)^y(D+H)^z}{\pi_0(A+E)^w(B+F)^x(C+G)^y(D+H)^z + (1-\pi_0)(I+M)^w(J+N)^x(K+O)^y(L+P)^z} \tag{1}$$

where A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, and P are probability multiples of each branch in the Baysian probability tree given in Figure 3 except π_t and 1-π_t, and w, x, y, and z denote the number of observed limit buy, market buy, limit sell, and market sell trader actions respectively.

The technical trader uses the probability of price going up as his probability of buying (i.e., $q_b^T = \pi_t$). As a result, buy and sell actions of the informed traders influence the q_b^T to go up and go down respectively. This model allows technical traders to carry forward information in past trading in order to generate information to take decisions on buying and selling.

When attempting to inflate or deflate the price and mislead the market, a manipulator uses the price behaviour in response to supply/demand, along with the behaviour of technical traders. The manipulation period is divided into three periods which we term as "ignition period", "momentum period" and "call-off period". In the ignition period (i.e., t_I time steps), the manipulator continues to buy with a probability q_b^I. In the momentum period (i.e., t_M time steps), the manipulator waits for the technical traders to raise the price further. Finally, in the call-off period, the manipulator sells his stocks for t_C period of time with probability 1 - q_b^C and exits from the market.

The trader composition in the pool set as 20% technical traders (n_T = 2), 70% liquidity traders (n_L = 7), and one manipulator. The M^* model with technical traders is simulated for t_T = 50000 time steps with parameter values p(0) = 1000, q_b = q_{lo} = 0.5, and π_0 = 0.5. In the technical trader belief model, it is assumed that when the price is to go up, informed traders are definitely buying (μ_1 = 1), and they definitely sell (μ_3 = 0) when the price is to go down. The model also assumes that the probabilities of submitting limit/market orders are equal (θ_1 = θ_2 = θ_3 = θ_4 = θ_5 = θ_6 = θ_7 = θ_8 = 0.5). In simulating the "pump and dump" manipulation, the manipulator parameters are set as: $t_S = 10000, t_I = t_M = t_C = 10000$, $q_b^I = 0.9$, and $q_b^C = 0.1$. The price offset (Δ) is drawn uniformly from a discrete set of random numbers from 1 to 4.

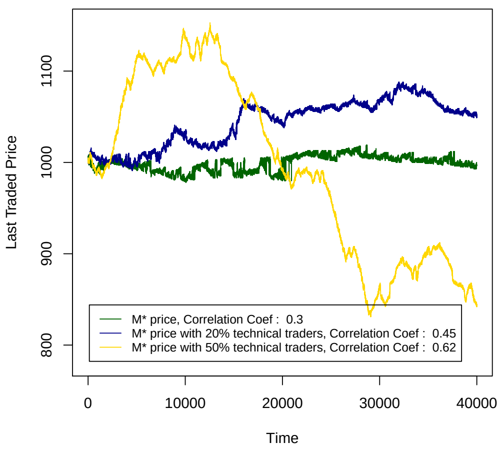

Figure 4. Price behaviour when technical traders are introduced to the M^* model

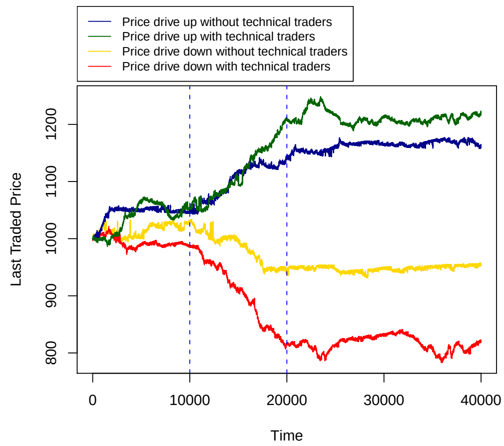

Figure 5. Price driving up/down with/without technical trading

The information content possessed by a technical trader depends on the past records that he/she could see when computing π_t. For simplicity we considered that all traders arrived in the market at the same time, and therefore they have the same set of information.

The behaviour of the model is analysed for different informed trader percentages (λ) in the Bayesian belief model. λ is assumed to be very low in a market (i.e., $0.001 < \lambda < 0.01$). The asymmetry between buying and selling is introduced to the technical trader belief model through the percentage of liquidity buying/selling (μ_2 and μ_4) and the resulting behaviour is analysed.

3 RESULTS AND DISCUSSION

As shown in Figure 4, the positive correlation in price increments increases as the percentage of technical traders increases. This observation indicates that technical trading adds a persistence behaviour to the Maslov prices. This momentum can be exploited by the manipulator in his "pump and dump" strategy.

In Figure 5, between the time steps 10000 and 20000, the manipulator performs buy/sell actions with a higher probability (0.75) to induce a supply and demand imbalance to drive the price up/down. It also shows how the momentum of technical trading amplifies the effect of this price change. Due to this momentum effect, the price continues to go up/down even after the manipulator stopped his continuous buying/selling at the 20000 time step.

When simulating the overall manipulation behaviour in this symmetric setting (i.e., $t_I = t_C$ and μ_2, μ_4 = 0.5), the average profits of liquidity traders, technical traders, and manipulator are observed to be zero. This means that the manipulator is not able to make any profits by selling immediately after raising the price. However, it is observed that if a purchase contains more information than a sale (i.e., liquidity buying percentage is less than the liquidity selling percentage or μ_2, μ_4 < 0.5), manipulation is possible. Figure 6 illustrates how the manipulator profit varies from positive to negative with the percentage of liquidity buying. From here onwards, when analysing the manipulator profits, we assume that the liquidity buying percentage (μ_2 and μ_4) is 0.25. This asymmetry leads our manipulator to buy

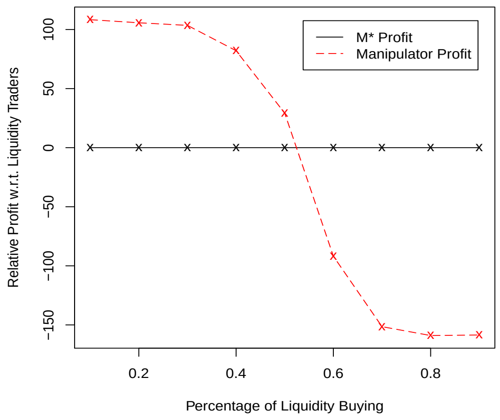

Figure 6. Manipulator profits for different liquidity buying percentages (μ_2/μ_4), λ = 0.005

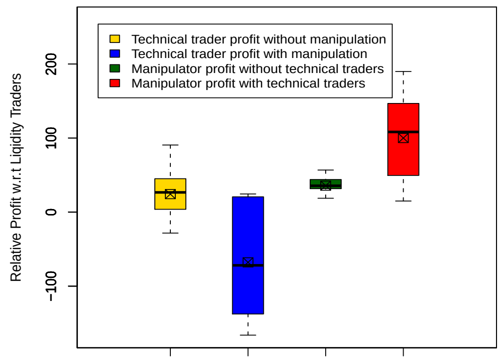

Figure 7. The effect of manipulation on trader profits

with a higher effect on prices and sell with little effect. This result confirms the finding of Allen and Gorton (1992).

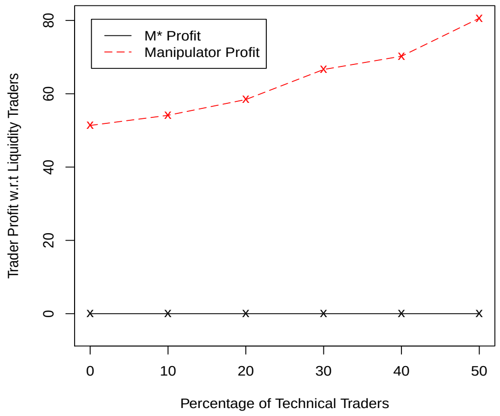

Figure 8. Manipulator profits with the percentage of technical traders (i.e., when changing n_T while n_T+n_L+1 is fixed at 10)

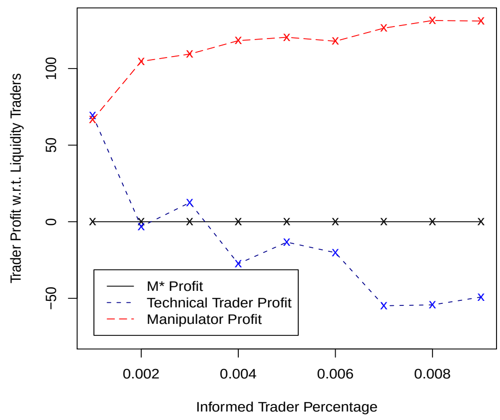

Figure 9. Manipulator profits for different informed trader percentages (λ) in the Bayesian belief model

Box plots in Figure 7 illustrate how manipulator profit increases with the presence of technical trading and how manipulation reduces the technical trader profit for 100 simulations. Figure 8 illustrates that the manipulator gets higher profits when the percentage of technical traders is high. This result confirms the findings of Aggarwal and Wu (2006).

The manipulator profit also depends on the belief model of the technical traders. As shown in Figure 9, if technical traders believe that there is a higher percentage of informed traders in the market (i.e., λ is high), a manipulator can easily pretend to be informed and hence his effort required to mislead the market is low.

As future work, we are planing to find evidence of real stock manipulation characteristics such as high volatility and low market efficiency in our manipulation model. This manipulation model could also be extended to introduce and study the dynamics of other price manipulation mechanisms such as “marking the close” and “painting the tape”. Moreover, when detecting the manipulation period, we plan to use metrics such as volatility (ARCH/GARCH effects), properties of price return distributions, and market efficiency measures such as the Hurst exponent.

4 Conclusion

A computational model is developed to characterise price manipulation in a limit order driven market. The Maslov (2000) limit order market model is extended to introduce technical traders and a manipulator to be used as a general framework for modelling manipulation strategies. We showed that a manipulator can pretend to be informed to mislead the technical traders in performing a "pump and dump" strategy in a limit order market. Concepts such as the role of supply and demand in determining price direction, how technical traders extract information from trading, and how manipulators use these principles to alter prices have been presented via the model properties. We also showed that the information asymmetry between buying and selling makes this manipulation possible. Technical traders lose money in the presence of manipulation and the manipulator profit is higher in the presence of technical trading. Moreover, if technical traders believe that there could be a higher percentage of informed traders in the market, the manipulator effort required to mislead the market is comparatively low.

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